A CAUSAL SKIN-EFFECT MODEL OF MICROSTRIP LINES

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Abstract

A complete transmission line model including skin-effect is derived to show effects of conductor skin loss on both the attenuation and phase factors. Frequency dependent behaviors of a microstrip line is also included in the model. Distortion of a short pulse due to the skin-effect including attenuation, dispersion, and propagation delay are investigated. Simulation results indicate that the skin-effect cannot be neglected in the phase factor in low conductivity transmission lines.

1. Introduction

A microstrip is a typical transmission line which is widely used in high frequency MIC's, MMIC's, and high speed digital VLSI's. Dispersion and attenuation are two important behaviors concerning signal propagation on microstrip lines. Dispersion is mainly due to the microstrip line's configuration, where fields are not confined within one dielectric medium. Attenuation loss is attributed to the skin-effect of conductors as well as lossy dielectric medium. Researches into the behaviors of various time domain signals have been reported in the past [1,2]. In reference [1], skin-effect was taken into consideration only while deriving expressions for the attenuation factor. Moreover, the skin loss was calculated using the static approximation formula found in [5,6]

Analysis of time domain responses including the skin-effect of coaxial lines dates back to the work done by Nahman *et al.* [3,4]. The inclusion of the skin-effect in the transmission line model generates effects on both the attenuation factor and the phase factor. The phase factor including the skin-effect causes a time delay in signal propagation. Although negligence of the skin-effect in the phase factor may not cause significant errors in low conductive loss microstrip lines, it becomes significant when resistivities of conductors are high. In practical cases, it is possible that a high resistivity interfacial layer may be formed between the relatively low resistivity conducting and dielectric materials due to the interaction of the materials during the line fabrication [8]. The conductivity of the interfacial layer then becomes dominant at high frequencies.

As an extension of the work in [1], analysis of a more accurate model is carried out in this paper. The model includes the skin-effect in the propagation factor, not only in the attenuation factor, but also in the phase factor in order to satisfy the causality requirement. The model also considers effects of frequency dependent behaviors of the microstrip line on the loss calculation. Simulations of propagation of time domain signals on a microstrip line will be conducted with the conductivity as a varying parameter. The simulation results will be compared with non causal model.

2. Microstrip Line Model and Formulation

(a) Effective dielectric constant and effective width:

A microstrip line is a dispersive transmission line because fields are not confined within one dielectric medium and distribution of fields around the conductor strip varies as operation frequency changes. To account for its dispersive behaviors, an effective width of the conductor strip and an effective dielectric constant were proposed [9]. The effective width and the effective dielectric constant are given by

$$\varepsilon_{re}(\omega) = \varepsilon_r - \frac{\varepsilon_r - \varepsilon_{re}(0)}{1 + (\omega/\omega_{pb})^2} \qquad (1)$$

and

$$W_e(\omega) = W + \frac{W_e(0) - W}{1 + (\omega/\omega_{pb})^2} \qquad (2)$$

where ε_r is the relative dielectric constant of dielectric medium, $\varepsilon_{re}(0)$ is the static effective dielectric constant, ω_{pb} is the cutoff frequency of the lowest order TE mode of the microstrip, W is the physical width of the conductor strip, and $W_e(0)$ is the static effective width of the conductor strip.

A main advantage of this model is that one can use the parallel plate model [11] by replacing the dielectric constant and physical width with the effective ones. Consequently, this model leads to frequency dependent inductance, capacitance, and characteristic impedance.

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(b) A causal lossy transmission line model:

A complete lossy transmission line model is shown in Figure (1). The propagation function γ in Laplace representation is given by

$$\gamma_{(s)} = \sqrt{Z_{(s)} Y_{(s)}} \quad (3)$$

where $Z_{(s)}$ and $Y_{(s)}$ are the series impedance and the shunt admittance per unit length of the transmission line, respectively. For the lossy dielectric material, $Y_{(s)}$ is given by

$$Y_{(s)} = sC + G \quad (4)$$

where C and G are capacitance and conductance per unit length, respectively. For the skin-effect of the lossy conductor under high frequency approximation, $Z_{(s)}$ is of the form [3]

$$Z_{(s)} = sL + K\sqrt{s} \quad (5)$$

where L and K are inductance per unit length and a current distribution factor of the skin-effect, respectively. The propagation function can be written as

$$\gamma_{(s)} = [(sL + K\sqrt{s})(sC + G)]^{1/2} \quad (6)$$

In the frequency domain, the skin-effect of Laplace representation $K\sqrt{s}$ reduces to $K\sqrt{j\omega}$, instead of $K\sqrt{\omega}$, as required by the causality of the physical circuit. Then, the propagation function in frequency domain $\gamma_{(j\omega)}$ is given by

$$\gamma_{(j\omega)} = [(j\omega L + R + jX)(j\omega C + G)]^{1/2} \quad (7)$$

where R=X, R and X are the real and imaginary parts of the series impedance due to the skin-effect, respectively.

Under small loss assumption, the propagation function can be expressed in terms of the loss factor $\alpha_{(\omega)}$ and the phase factor $\beta_{(\omega)}$

$$\gamma_{(j\omega)} = \alpha_{(\omega)} + j\beta_{(\omega)} \quad \text{(8-a)}$$

where:

$$\beta_{(\omega)} \equiv \omega\sqrt{L_{(\omega)} C_{(\omega)}} = \frac{\omega}{v}\sqrt{\varepsilon_{re(\omega)}} + \frac{1}{2}\frac{R_{(\omega)}}{Z_{o(\omega)}} \quad \text{(8-b)}$$

v is the speed of light,

$$\alpha = \alpha_c + \alpha_d = \frac{1}{2}\frac{R_{(\omega)}}{Z_{o(\omega)}} + \frac{1}{2} G Z_{o(\omega)} \quad \text{(8-c)}$$

$Z_{0(\omega)} = \sqrt{L_{(\omega)}/C_{(\omega)}}$ is characteristic impedance.

It can be seen that the conductor loss including the skin-effect has effects on not only the loss factor, but also the phase factor. The causality of a physical circuit introduces the skin-effect equivalent impedance as $R + jX$, instead of R alone.

(c) Frequency dependence of conductor loss:

There are various methods to calculate the skin-effect. The work done by Pucel *et al.* [5,6] was based on the "incremental inductance rule" introduced by Wheeler [7]. In reference [1], Leung and Balanis used Pucel's formula to calculate conductor loss. It can be recognized that the Pucel's formula is valid only under static case because static characteristic impedance is used in their formula.

As mention above, the characteristic impedance is frequency dependent in microstrip lines. The derived equation (8)'s indicate that the frequency dependent characteristic impedance should be used in the formula. If the skin-effect is written in the form of

$$R = X = K\sqrt{\omega} \quad (9)$$

then the constant K is given by

$$K = \frac{1}{\sqrt{2}\,\pi h}\sqrt{\frac{\mu}{\sigma}}\,[1 - (\frac{w'}{4h})^2]\{1 + \frac{h}{w'} + \frac{h}{\pi w'}[\ln(\frac{2h}{T}) - \frac{T}{w}]\} \quad (10)$$

for $1/2\pi \le w/h \le 2$, where μ is the permeability of the conductor, σ is the conductivity of the conductor, T is thickness of the conductor strip, h is the height of the dielectric substrate, w is the width of the conductor strip, and w' is given by

$$w' = w + \frac{T}{\pi}[\ln(\frac{2h}{T}) + 1] \quad (11)$$

(d) time domain signal propagation:

The distorted time domain waveforms are computed using the expression

$$V_{(t,z=l)} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} V_{(\omega,z=0)} \exp(j\omega t - \gamma_{(j\omega)} l)\, d\omega \quad (12)$$

where l is the distance at which the distorted waveforms are observed and $V_{(\omega,z=0)}$ is the Fourier transform of the input pulse at the reference point.

3. Simulation Examples and Results

To illustrate how the skin-effect affects the time domain signal, a ramp pulse and a Gaussian pulse propagating on a microstrip line are simulated using previous described model. For simplicity, the dielectric loss is not included in the following simulation, which will not illustratively weaken the purpose of this paper.

The configuration of the microstrip line to be simulated is shown in Figure (2). The static parameters such as static effective dielectric constant, effective width, and characteristic impedance are calculated using formula given in [10]. The calculated static characteristic impedance is 62.6 Ω.

The conductivity σ is used as a variable parameter changing from 10^6 to 10^7 S/m in the simulations in order to see the effects of the skin-effect. Phase factors as functions of frequency as well as conductivity, normalize to non-dispersion phase factor ω/v, are shown in Figure(3). Comparing with the phase factor without considering the causality, one can find out that the skin-effect impedance mainly affects the phase factors in relatively low frequency range. The attenuation factors are shown in Figure (4).

Figure (5-6) show the simulation results of the distorted waveforms of a Gaussian pulse with 20 picosecond full duration half magnitude (FDHM) as the conductivity varies from 10^6 to 10^7 S/m. The results of a step-like ramp pulse, which has a ramp duration of 20ps are shown in Figure (7-8). For comparison, the simulations using formula in [1] are plotted in the same format. It can be seen that the time domain waveforms are delayed more when the skin-effect is included in the phase factor. When the conductivity $\sigma=10^7$ S/m, the time delay due to the skin-effect becomes significant. In the case of the conductivity $\sigma = 10^6$ S/m, the delay time is about 7 ps. For high speed signal transmission, this time delay cannot be ignored.

4. Conclusion

Complete formulation of the transmission function including the skin-effect is presented. The model also includes frequency dependent characteristic impedance. Simulations of step-like ramp pulse and gaussian pulse propagating on a microstrip line with conductivity as a varying parameter are conducted. The simulation results indicate that the time delay is significant in the low conductivity materials and therefore more accurate formula should be used.

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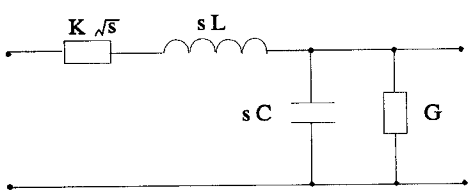

Figure 1: Lossy transmission line model.

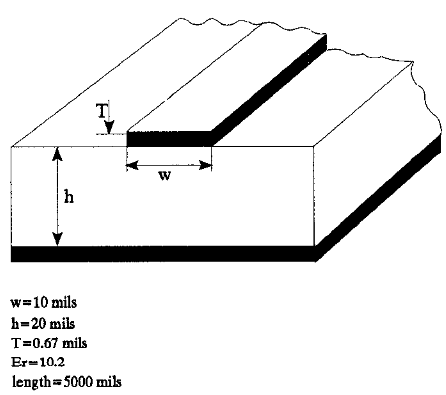

Figure 2: A microstrip line for simulation.

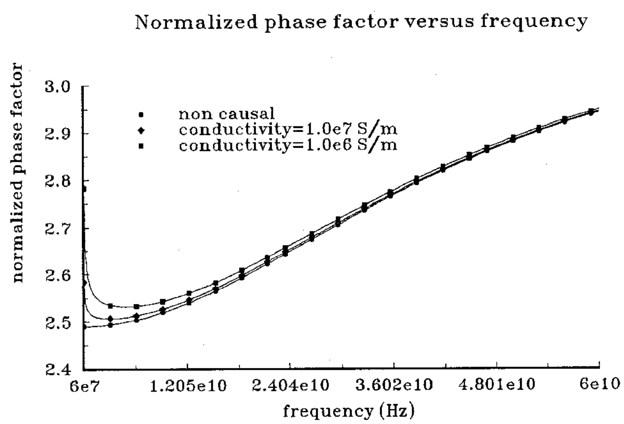

Figure 3: Normalized phase factor versus frequency.

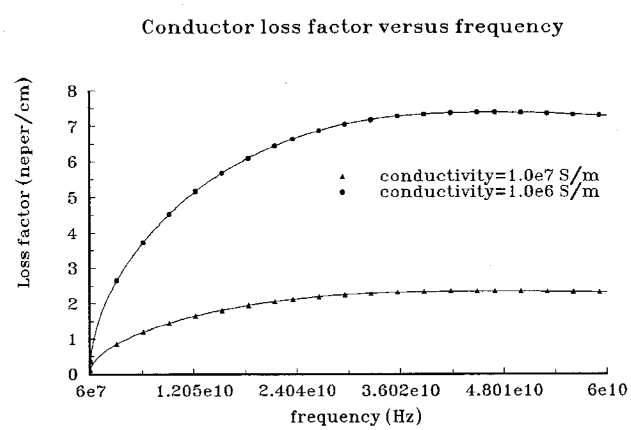

Figure 4: Loss versus freuqency.

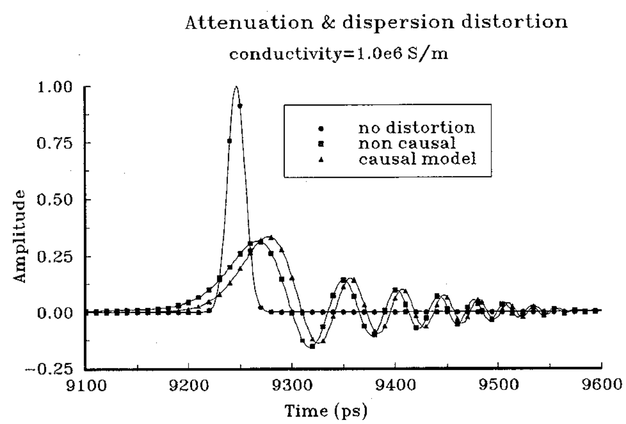

Figure 5: Gaussian pulse distortion for σ=1.0e6 S/m.

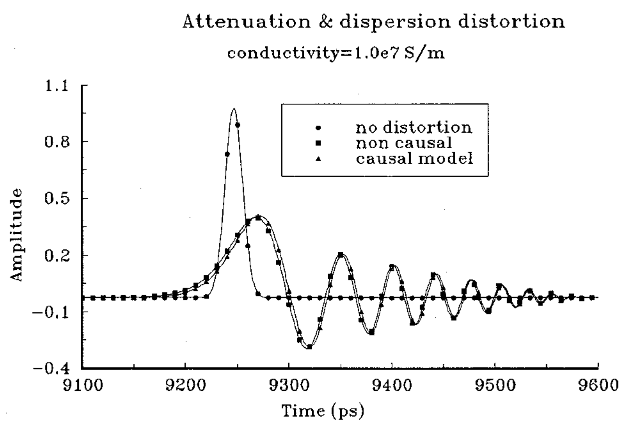

Figure 6: Gaussian pulse distortion for σ=1.0e7 S/m.

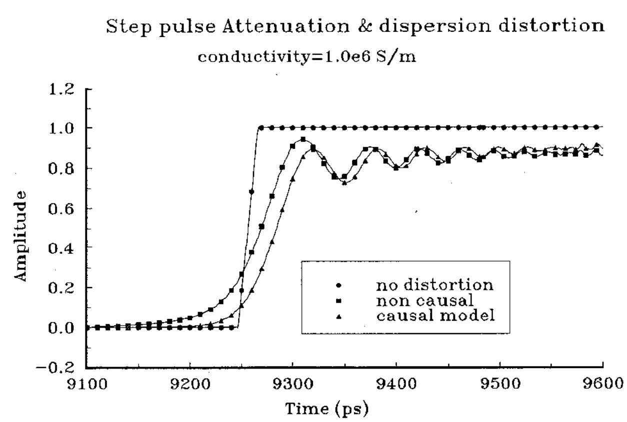

Figure 7: Step-like ramp pulse distortion for σ=1.0e6 S/m.

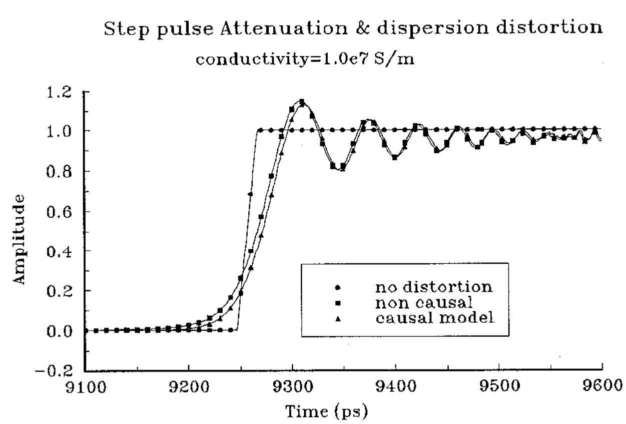

Figure 8: Step-like ramp pulse distortion for σ=1.0e7 S/m.